

Entanglement, Joint Measurement, and State Reduction

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Entanglement is perhaps the most important new feature of the quantum world. It is expressed in quantum theory by the joint measurement formula. We prove the formula for projection valued observables from a plausible assumption, which for spacelike separated measurements is a consequence of causality. State reduction is simply a way to express the joint measurement formula after one measurement has been made, and its result known.

KEY WORDS: entanglement; joint measurement; state reduction; causality; measurement problem.

1. INTRODUCTION

Entanglement is perhaps the most important new feature of the quantum world. It is expressed in quantum theory by the joint measurement formula (JMF). I prove that the JMF is equivalent to the conjunction of two assumptions. One is NOEFFECT: A nonselective measurement of one member of a pair of entangled noninteracting systems has no effect on measurement probabilities for the other member. (The measurement is *nonselective* if we do not use its result to condition measurement probabilities for the other member.)

For projection valued observables, the JMF is equivalent to NOEFFECT alone. An example shows that for general observables, NOEFFECT $\not\Rightarrow$ JMF.

A violation of NOEFFECT in spacelike separated measurements would allow superluminal communication. Thus causality implies the JMF for spacelike separated measurements of projection valued observables. The JMF implies violations Bell's inequality, and thus violations of locality. Thus, within the quantum formalism, *causality implies nonlocality*.

"No signaling" theorems have eliminated the worry that the nonlocality in quantum theory violates causality (Jordan, 1983; Zanchini and Barletta, 1991).

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Our result shows that not only does nonlocality not violate causality, it is required to preserve causality.

We prove that the state reduction formula (SRF) is an immediate corollary of the JMF: state reduction is simply a way to express the JMF after one measurement has been made, and its result known. We then prove the von Neumann–Lüders projection postulate from the SRF. Thus the “postulate” is a theorem, a consequence of the JMF.

All this sheds new light on entanglement, joint measurement, state reduction, nonlocality, and causality in quantum theory.

The paper is organized as follows. Section 2 reviews the postulates of quantum theory, without the JMF or SRF. Section 3 describes my approach to the JMF and SRF. Section 4 describes Masanao Ozawa’s approach to the JMF and SRF and compares our two approaches. Section 5 argues that there is no measurement problem. Section 6 gives the example showing that NOEFFECT $\not\Rightarrow$ JMF.

2. QT-

To prepare for a discussion of the JMF and SRF, we review the postulates of quantum theory, excluding the JMF and SRF. We call the theory *QT-*. For more details, see Kraus (1983) and Busch *et al.* (1991).

A *quantum system* \mathbf{S} is represented by a complex Hilbert space $H_{\mathbf{S}}$, which in this paper will be finite dimensional. A *preparation* of \mathbf{S} is represented by a *state*, a density operator σ on $H_{\mathbf{S}}$. A *measurement* of \mathbf{S} is represented by an *observable*, a positive operator valued measure (POVM) \mathcal{S} . Let \mathcal{S} map the measured value s to E_s , $0 \leq E_s \leq \mathbf{I}$. According to the *measurement formula*, the probability of result s for an \mathcal{S} measurement on state σ is $\text{Pr}(s) = \text{Tr}(E_s \sigma)$.

If \mathbf{S} is isolated, then σ evolves unitarily according to *Schrödinger’s equation*: $\sigma \rightarrow U_{\mathbf{S}} \sigma U_{\mathbf{S}}^\dagger$. Important: for now, “isolated” excludes “entangled with another system.” The extent to which Schrödinger’s equation applies to a quantum system entangled with another will be the focus of Section 4.

Let \mathbf{P} be another quantum system. Then $\mathbf{S} + \mathbf{P}$ is represented by $H_{\mathbf{S}} \otimes H_{\mathbf{P}}$. Thus the states τ of $\mathbf{S} + \mathbf{P}$ are density operators on $H_{\mathbf{S}} \otimes H_{\mathbf{P}}$, and the observables are POVMs whose values are positive operators on $H_{\mathbf{S}} \otimes H_{\mathbf{P}}$. A measurement of \mathcal{S} on $\mathbf{S} + \mathbf{P}$ is represented by the POVM that maps s to $E_s \otimes \mathbf{I}$. Then from the measurement formula, $\text{Pr}(s) = \text{Tr}[(E_s \otimes \mathbf{I})\tau]$. The systems \mathbf{S} and \mathbf{P} *do not interact* if the unitary evolution operator of $\mathbf{S} + \mathbf{P}$ factors: $U_{\mathbf{S}+\mathbf{P}} = U_{\mathbf{S}} \otimes U_{\mathbf{P}}$.

If for some state σ , $\text{Pr}(s) = \text{Tr}(E_s \sigma)$ for every observable \mathcal{S} and every result s , then σ is the state of \mathbf{S} . For the $\text{Tr}(E_s \sigma)$ uniquely determine the state σ . We say that “probabilities determine states.”

For reference we list several identities that we will use without comment: $\text{Tr}(XY) = \text{Tr}(YX)$, $\langle s_1, \otimes p_1 | s_2 \otimes p_2 \rangle = \langle s_1 | p_1 \rangle \langle s_2 | p_2 \rangle$, $X \otimes Y = (X \otimes \mathbf{I})(\mathbf{I} \otimes Y)$, and $(X \otimes Y)|s \otimes p\rangle = X|s\rangle \otimes Y|p\rangle$. The *partial trace* operator Tr_p maps

operators on $\mathbf{S} + \mathbf{P}$ to operators on \mathbf{S} (Cohen-Tannoudji *et al.*, 1997). We have the partial trace identities $\text{Tr}(X) = \text{Tr}[\text{Tr}_{\mathbf{P}}(X)]$ and $\text{Tr}_{\mathbf{P}}[(X \otimes \mathbf{I})Y] = X\text{Tr}_{\mathbf{P}}(Y)$ (Busch *et al.*, 1991; Kraus, 1983). Using these identities and “probabilities determine states,” we see that if the state of $\mathbf{S} + \mathbf{P}$ is τ , then the state of \mathbf{S} is $\text{Tr}_{\mathbf{P}}(\tau)$:

$$\Pr(s) = \text{Tr}[(E_s \otimes \mathbf{I})\tau] = \text{Tr}\{\text{Tr}_{\mathbf{P}}[(E_s \otimes \mathbf{I})\tau]\} = \text{Tr}[E_s \text{Tr}_{\mathbf{P}}(\tau)]. \quad (1)$$

3. JOINT MEASUREMENT AND STATE REDUCTION

In this section we prove results about joint measurement, state reduction, causality, and nonlocality in the theory QT- defined in Section 2.

3.1. Joint Measurement Formula

Prepare $\mathbf{S} + \mathbf{P}$ in state τ at time t_1 , after which \mathbf{S} and \mathbf{P} do not interact. At time $t_{\mathbf{P}} \geq t_1$ measure observable \mathcal{P} of \mathbf{P} , with result p . At time $t_{\mathbf{S}} \geq t_1$ measure observable \mathcal{S} of \mathbf{S} , with result s . Let $U_{\mathbf{P}}$ be the unitary evolution operator for \mathbf{P} from t_1 to $t_{\mathbf{P}}$. Let $U_{\mathbf{S}}$ be the unitary evolution operator for \mathbf{S} from t_1 to $t_{\mathbf{S}}$. Then

$$\Pr(s \& p) = \text{Tr}[(U_{\mathbf{S}}^\dagger E_s U_{\mathbf{S}} \otimes U_{\mathbf{P}}^\dagger E_p U_{\mathbf{P}})\tau]. \quad (\text{JMF})$$

For given $t_{\mathbf{P}}$, \mathcal{P} , $t_{\mathbf{S}}$, and \mathcal{S} let the POVM representing the joint measurement map the result (s, p) to $E_{s \& p}$. Then according to the measurement formula, $\Pr(s \& p) = \text{Tr}(E_{s \& p} \tau)$ for all s, p , and τ . Thus the JMF for the measurement is equivalent to

$$\forall s, p \ E_{s \& p} = U_{\mathbf{S}}^\dagger E_s U_{\mathbf{S}} \otimes U_{\mathbf{P}}^\dagger E_p U_{\mathbf{P}}. \quad (2)$$

The (nonselective) probability of s is $\sum_p \Pr(s \& p) = \text{Tr}[(\sum_p E_{s \& p})\tau]$. If the \mathcal{P} measurement is not made, then according to Eq. (5), the probability of s is

$$\text{Tr}[E_s(U_{\mathbf{S}}\text{Tr}_{\mathbf{P}}(\tau)U_{\mathbf{S}}^\dagger)] = \text{Tr}[(U_{\mathbf{S}}^\dagger E_s U_{\mathbf{S}} \otimes \mathbf{I})\tau].$$

NOEFFECT from Section 1 asserts that the two probabilities are equal:

A nonselective measurement of one member of a pair of entangled noninteracting systems has no effect on measurement probabilities for the other member.

Thus according to NOEFFECT,

$$\forall s \ \sum_p E_{s \& p} = U_{\mathbf{S}}^\dagger E_s U_{\mathbf{S}} \otimes \mathbf{I}. \quad (\text{NOEFFECT})$$

Similarly,

$$\forall p \ \sum_s E_{s \& p} = \mathbf{I} \otimes U_{\mathbf{P}}^\dagger E_p U_{\mathbf{P}}. \quad (\text{NOEFFECT})$$

Consider also the assertion that $E_{s \& p}$ is the product of its marginals:

$$\forall s, p \quad E_{s \& p} = \left(\sum_p E_{s \& p} \right) \left(\sum_s E_{s \& p} \right). \quad (\text{PRODMARG})$$

Theorem 1. *For given $t_p, \mathcal{P}, t_s,$ and $S,$*

$$\text{JMF} \iff (\text{NOEFFECT} + \text{PRODMARG}).$$

Proof: We use the JMF in the form Eq. (2).

JMF \Rightarrow NOEFFECT. Sum Eq. (2) over p and use $\sum_p E_p = \mathbf{I}$. (This is the no signaling theorem of Jordan, 1983.)

JMF \Rightarrow PRODMARG. Multiply the two NOEFFECT equations, which we have just shown follow from the JMF, and use Eq. (2) to obtain PRODMARG.

(NOEFFECT + PRODMARG) \Rightarrow JMF. Multiply the two NOEFFECT equations and use PRODMARG to obtain Eq. (2). \square

Corollary 1. *If \mathcal{P} and S are projection valued, then JMF \iff NOEFFECT.*

Proof: From the theorem, it is sufficient to prove that if \mathcal{P} and S are projection valued, then NOEFFECT \Rightarrow PRODMARG. For a projection valued $S,$ the E_s are orthogonal projections. Thus the $U_s^\dagger E_s U_s \otimes \mathbf{I}$ on the right side of the first NOEFFECT equation are orthogonal projections. Sums of these projections are projections. Every POVM on a product space with projection valued marginal measures satisfies PRODMARG (Davies, 1976, Th. 2.1, Eq. (2.7)). \square

The example E' of Section 6 shows that for general POVMs, NOEFFECT $\not\Rightarrow$ JMF.

The implication NOEFFECT \Rightarrow JMF for projection valued observables is of special interest. As noted in Section 1, for spacelike separated measurements causality implies NOEFFECT. Thus, in QT-:

Corollary 2. *Causality implies the JMF for spacelike separated measurements of projection valued observables.*

The JMF predicts violations of Bell’s inequality for some spacelike separated measurements of projection valued observables. It thus predicts violations of locality. Thus, in QT-:

Corollary 3. *Causality implies nonlocality.*

We now turn to the SRF. Since probabilities determine states, we can reformulate NOEFFECT:

A nonselective measurement of one member of a pair of entangled noninteracting systems has no effect on the state of the other member.

But the SRF says that if we make a *selective* measurement, conditioning the state of \mathbf{S} on the \mathcal{P} measurement result, then we must *reduce* the state of \mathbf{S} .

3.2. State Reduction Formula

Prepare $\mathbf{S} + \mathbf{P}$ in state τ at time t_1 , after which \mathbf{S} and \mathbf{P} do not interact. At t_1 measure observable \mathcal{P} of \mathbf{P} , with result p . Let $U_{\mathbf{S}}$ be the unitary evolution operator of \mathbf{S} over the time of the \mathcal{P} measurement. Let σ_p be the state of \mathbf{S} after the \mathcal{P} measurement, conditioned on p . Then

$$\sigma_p = U_{\mathbf{S}} \frac{\text{Tr}_{\mathbf{P}}[(\mathbf{I} \otimes E_p)\tau]}{\text{Tr}[(\mathbf{I} \otimes E_p)\tau]} U_{\mathbf{S}}^\dagger. \quad (\text{SRF})$$

Remarks. (i) The SRF requires *no* assumptions about the state of \mathbf{P} after the \mathcal{P} measurement, even that \mathbf{P} still exists. (ii) Since we do not assume that Schrödinger's equation applies to a system entangled with another, we cannot interpret the SRF as giving the evolution of \mathbf{S} during the \mathcal{P} measurement. (iii) It is *classical* information, i.e., p , which allows us to reduce the state of \mathbf{S} to σ_p . (iv) From the SRF, $\sum_p \text{Pr}(p)\sigma_p = U_{\mathbf{S}} \text{Tr}_{\mathbf{P}}(\tau) U_{\mathbf{S}}^\dagger$, the unreduced state.

Theorem 2. JMF \Rightarrow SRF.

Proof: Measure \mathcal{S} immediately after the \mathcal{P} measurement. From the JMF,

$$\text{Pr}(s \& p) = \text{Tr}\{(U_{\mathbf{S}}^\dagger E_s U_{\mathbf{S}} \otimes E_p)\tau\}. \quad (3)$$

Thus for every \mathcal{S} and every s ,

$$\begin{aligned} \text{Pr}(s|p) &= \frac{\text{Pr}(s \& p)}{\text{Pr}(p)} = \frac{\text{Tr}\{(U_{\mathbf{S}}^\dagger E_s U_{\mathbf{S}} \otimes E_p)\tau\}}{\text{Tr}[(\mathbf{I} \otimes E_p)\tau]} \\ &= \frac{\text{Tr}\{\text{Tr}_{\mathbf{P}}[(U_{\mathbf{S}}^\dagger E_s U_{\mathbf{S}} \otimes \mathbf{I})(\mathbf{I} \otimes E_p)\tau]\}}{\text{Tr}[(\mathbf{I} \otimes E_p)\tau]} \\ &= \text{Tr} \left\{ E_s \left(U_{\mathbf{S}} \frac{\text{Tr}_{\mathbf{P}}[(\mathbf{I} \otimes E_p)\tau]}{\text{Tr}[(\mathbf{I} \otimes E_p)\tau]} U_{\mathbf{S}}^\dagger \right) \right\}. \end{aligned} \quad (4)$$

Since probabilities determine states, the SRF follows. \square

(For more on this kind of reasoning to obtain state reduction, see Svetlichny, 2002.)

Conversely, given the SRF, a rearrangement of Eq. (4) proves Eq. (3). Thus *State reduction is simply a way to express the JMF after one measurement has been made, and its result known.*

K. Kraus makes a similar statement: “[State reductions] provide a convenient ‘shorthand’ description of correlation measurements. We may thus conclude that, contrary to widespread belief, [state reductions] can be perfectly well understood, *if* quantum mechanics is assumed to be valid also for measuring instruments.” (Kraus, 1983, p. 99; my emphasis.) Our proof of the SRF does not assume that quantum mechanics is valid for measuring instruments. Thus Kraus’ *if* clause is unnecessary. For more on this, see Section 5.

Corollary 4. *If \mathcal{P} is projection valued, then NOEFFECT \Rightarrow SRF.*

Proof: Measure a projection valued observable \mathcal{S} immediately after the \mathcal{P} measurement. Then Corollary 2 implies Eq. (3), which implies Eq. (4) for projections E_s , which is sufficient to imply the SRF for the \mathcal{P} measurement. \square

We close this section with a discussion of the von Neumann–Lüders measurement model. Let \mathbf{S} be a quantum *system* to be measured and \mathbf{P} be a quantum *probe*, which is part of a macroscopic measuring apparatus. Initially \mathbf{S} and \mathbf{P} are separated and unentangled, and in states σ_0 and π_0 . The system enters the measuring apparatus, interacts with the probe, and leaves the apparatus. Let $\tau = U(\sigma_0 \otimes \pi_0)U^\dagger$ be the state of $\mathbf{S} + \mathbf{P}$ after the interaction, which is called a *premeasurement*. (A premeasurement is *not* a measurement: a premeasurement is reversible and no measured value is created.) Now measure \mathcal{P} , with the result p appearing on the measuring apparatus. In the von Neumann–Lüders model, the \mathcal{P} measurement serves as a proxy for an \mathcal{S} measurement.

The model is for projection valued \mathcal{S} with an associated self-adjoint operator $\sum_{ij} s_i |s_{ij}\rangle\langle s_{ij}|$. Let \mathcal{P} be a nondegenerate projection valued observable with an associated self-adjoint operator $\sum_i p_i |p_i\rangle\langle p_i|$. Choose a unitary operator U with $U(|s_{ij}\rangle|p_0\rangle) = |s_{ij}\rangle|p_i\rangle$ for some fixed initial state $|p_0\rangle$ of \mathbf{P} . Then for an initial vector state $|s_0\rangle = \sum_{ij} a_{ij} |s_{ij}\rangle$ of \mathbf{S} , $U(|s_0\rangle|p_0\rangle) = \sum_{ij} a_{ij} |s_{ij}\rangle|p_i\rangle \equiv |t\rangle$. For a \mathcal{P} measurement on state $|t\rangle$, $\Pr(p_k) = \sum_j |a_{kj}|^2$. For an \mathcal{S} measurement on state $|s_0\rangle = \Pr(s_k)$ has the same value. Thus a \mathcal{P} measurement on state $|t\rangle$ with result p_k is also an \mathcal{S} measurement on state $|s_0\rangle$ with result s_k .

The SRF gives the reduced state σ_{s_k} of \mathbf{S} after the \mathcal{S} measurement. To apply it, we first use the identity $\text{Tr}_{\mathbf{P}}[(\mathbf{I} \otimes X)Y] = \text{Tr}_{\mathbf{P}}[Y(\mathbf{I} \otimes X)]$ (Kraus, 1983, Eq. (5.15)):

$$\begin{aligned} \text{Tr}_{\mathbf{P}}\{(\mathbf{I} \otimes E_{p_k})\tau\} &= \text{Tr}_{\mathbf{P}}\{(\mathbf{I} \otimes E_{p_k})|t\rangle\langle t|(\mathbf{I} \otimes E_{p_k})\} \\ &= \text{Tr}_{\mathbf{P}}\{\sum_j a_{kj} |s_{kj}\rangle|p_k\rangle \sum_j \bar{a}_{kj} \langle s_{kj}| \langle p_k|\} \end{aligned}$$

$$\begin{aligned}
&= \text{Tr}_{\mathbf{P}} \{ E_{s_k} |s_0\rangle |p_k\rangle \langle s_0| E_{s_k} \langle p_k| \} \\
&= E_{s_k} |s_0\rangle \langle s_0| E_{s_k}.
\end{aligned}$$

Substitute this into the SRF:

$$\sigma_{s_k} = U_{\mathbf{S}} \frac{E_{s_k} |s_0\rangle \langle s_0| E_{s_k}}{\text{Tr} \{ E_{s_k} |s_0\rangle \langle s_0| E_{s_k} \}} U_{\mathbf{S}}^\dagger.$$

As a vector, the reduced state is $U_{\mathbf{S}} E_{s_k} |s_0\rangle / \|E_{s_k} |s_0\rangle\|$. This is the state given by the *von Neumann–Lüders projection postulate*. Since $\text{JMF} \Rightarrow \text{SRF}$, the “postulate” is a *theorem* of QT- + JMF.

4. OZAWA’S APPROACH

Masanao Ozawa has published several papers on joint measurement and state reduction (Ozawa, 1997a,b, 1998a,b, 2000a,b,c). He argues, correctly I believe, that existing proofs of the JMF and SRF are inadequate or flawed (Ozawa, 1997a, p. 233; 1997b, p. 123; 1998a, p. 616; 2000a, p. 6). He then offers his own proofs of the JMF (Ozawa, 1997a, Th. 5.1; 2000a, Th. 3) and the SRF (Ozawa, 1997b, Eq. (43); 1998a, Eq. (32)). Ozawa considers projection valued observables only.

As emphasized in Section 2, QT- does not assume that Schrödinger’s equation applies to a quantum system entangled with another. But we can prove:

A unitary evolution of one member of a pair of entangled noninteracting systems has no effect on the state of the other member.

Proof: Since \mathbf{S} and \mathbf{P} do not interact, the unitary evolution operator of $\mathbf{S} + \mathbf{P}$ factors: $V_{\mathbf{S}+\mathbf{P}} = V_{\mathbf{S}} \otimes V_{\mathbf{P}}$. Let τ be the initial state of $\mathbf{S} + \mathbf{P}$. Then for all E_s ,

$$\begin{aligned}
\text{Tr}[E_s(V_{\mathbf{S}}\text{Tr}_{\mathbf{P}}(\tau)V_{\mathbf{S}}^\dagger)] &= \text{Tr}[(E_s \otimes \mathbf{I})(V_{\mathbf{S}} \otimes \mathbf{I})\tau(V_{\mathbf{S}}^\dagger \otimes \mathbf{I})] \\
&= \text{Tr}[(E_s \otimes \mathbf{I})(\mathbf{I} \otimes V_{\mathbf{P}}^\dagger)(\mathbf{I} \otimes V_{\mathbf{P}})(V_{\mathbf{S}} \otimes \mathbf{I})\tau(V_{\mathbf{S}}^\dagger \otimes \mathbf{I})] \\
&= \text{Tr}[(E_s \otimes \mathbf{I})(V_{\mathbf{S}} \otimes V_{\mathbf{P}})\tau(V_{\mathbf{S}} \otimes V_{\mathbf{P}})^\dagger] \quad (5) \\
&= \text{Tr}\{E_s \text{Tr}_{\mathbf{P}}[(V_{\mathbf{S}} \otimes V_{\mathbf{P}})\tau(V_{\mathbf{S}} \otimes V_{\mathbf{P}})^\dagger]\}.
\end{aligned}$$

Thus the state of \mathbf{S} at a later time, $\text{Tr}_{\mathbf{P}}[(V_{\mathbf{S}} \otimes V_{\mathbf{P}})\tau(V_{\mathbf{S}} \otimes V_{\mathbf{P}})^\dagger]$, is the same as the state given by Schrödinger’s equation applied to \mathbf{S} alone, $V_{\mathbf{S}}\text{Tr}_{\mathbf{P}}(\tau)V_{\mathbf{S}}^\dagger$. \square

(This is the no signaling theorem of Zanchini and Barletta, 1991, Th. 3.)

For projection valued observables, we proved the JMF in Corollary 2 and the SRF in Corollary 6 from the assumption NOEFFECT:

A nonselective measurement of one member of a pair of entangled noninteracting systems has no effect on the unreduced state of the other member.

(We use the reformulated version following Corollary 4 and add the word “unreduced” for clarity and comparison.)

Ozawa uses a different assumption:

A selective measurement of one member of a pair of entangled noninteracting systems has no effect on the reduced state of the other member.

(In Ozawa, 1998a see the discussions surrounding Eqs. (5), (6), and (15), and also p. 622.)

One example of Ozawa’s use of his assumption is in his proof of the JMF in Ozawa (1997a, Th. 5.1), when passing from the third to the fourth member in the equation between Eqs. (9) and (10). (Ozawa has confirmed this reading in a private communication.) Another example is in his proof of the SRF in Ozawa (1998a, Sec. 7).

Ozawa agrees that the SRF gives the reduced state σ_p after the \mathcal{P} measurement, but his assumption rules out our view that the reduction occurs with the measurement, a view he rejects (Ozawa, 1997b, p. 123). For him, the reduction occurs earlier, with the *premeasurement*, to a state that we denote σ_p^1 . (σ_p^1 is denoted $\rho(t + \Delta t|a(t) \in \{p\})$ in Ozawa, 2000a, and $\rho(t + \Delta t|p)$ in Ozawa, 1997a, 1998b.) (Warning: Ozawa sometimes calls just the premeasurement—which he calls *stage I*—a “measurement” (Ozawa, 1997b, Eq. (1); 1998a, Eq. (1))).

According to Ozawa, σ_p^1 is the state of \mathbf{S} after the premeasurement, “conditional upon” the results p of the later \mathcal{P} measurement (Ozawa, 2000a, p. 9), or “that leads to the outcome p ” in the measurement (Ozawa, 1997b, p. 124). More specifically:

Suppose the system and probe are spin- $\frac{1}{2}$ particles brought into the singlet state by the premeasurement. After the premeasurement is complete, we can choose to measure the spin of the probe in the z -direction or the x -direction. If we choose the z -direction and the result is “up,” then the system was prepared in the “down” eigenstate σ_\downarrow^1 just after the premeasurement. If we choose the x -direction and the result is “left,” then the system was prepared in the “right” eigenstate σ_\rightarrow^1 , just after the premeasurement. [Private communication.]

If, according to Ozawa’s assumption, \mathbf{S} evolves unitarily from after the premeasurement until after the probe measurement, and if its state after the probe measurement is σ_p , then its state after the premeasurement is, from the SRF,

$$\frac{\text{Tr}_P[(\mathbf{I} \otimes E_p)\tau]}{\text{Tr}[(\mathbf{I} \otimes E_p)\tau]}.$$

This is Ozawa’s expression for σ_p^1 (Ozawa, 1997b, Eq. (34); 1998a, Eq. (32)). For him, the SRF describes a unitary evolution of \mathbf{S} from σ_p^1 to σ_p . For me, the SRF does not describe an evolution of \mathbf{S} , as stated in the remarks following the SRF.

Bell’s inequality is relevant here. The inequality shows that not only is the result p of the probe measurement not known before the measurement, *it does not*

exist before the measurement. This even though p would be correlated with the result of a latter measurement of \mathcal{S} . Mermin explains this clearly (Mermin, 1981, 1985).

For me, this makes the states σ_p^1 problematic. Furthermore, they are not needed to obtain the SRF: we proved in Section 3 that the correlations given by the JMF imply that the state of \mathbf{S} after the \mathcal{P} measurement is given by the SRF. State reduction is not a *dynamical* consequence of Schrödinger's equation; it is a *logical* consequence of entanglement.

To reject attributing the state reduction of \mathbf{S} to the \mathcal{P} measurement is to cling to classical notions of causality, instead of fully embracing that remarkable new quantum phenomenon, entanglement.

5. THE MEASUREMENT PROBLEM

We have been careful to distinguish the probe \mathbf{P} , a *quantum* system, from the macroscopic apparatus measuring it. We made no assumptions about the apparatus other than the minimal requirement that it display measurement results in accordance with the measurement formula. In particular, we did not model it as a quantum system obeying Schrödinger's equation. Modeling the apparatus in this way leads to the notorious *measurement problem*: The appearance of a definite measured value on the apparatus would be a state reduction of the apparatus, which is inconsistent with Schrödinger's equation.

I argue at length elsewhere that the apparatus cannot be so modeled and thus there is no measurement problem (Macdonald, 2002). Here I support this point of view only with the following quotes.

In *The Quantum Theory of Measurement*, P. Busch, and P. Lahti, and P. Mittelstaedt write, "The quantum theory of measurement is motivated by the idea of the universal validity of quantum mechanics, according to which this theory should be applicable, in particular, to the measuring process. One would expect, and most researchers in the foundations of quantum mechanics, have done so, that the problem of measurement should be solvable *within* quantum mechanics. The long history of this problem shows that . . . there seems to be no straightforward route to its solution" (Busch *et al.*, 1991, p. 138).

K. Kraus also describes the measuring apparatus as a quantum system (Kraus, 1983, pp. 81, 99). But "There are good reasons to doubt that quantum mechanics in its present form is the appropriate theory of macroscopic systems" (Kraus, 1983, p. 100).

According to A. Leggett, "What is required is to explain how one particular macrostate can be forced by the quantum formalism to be realized. In the opinion of the present author (which is shared by a small but growing minority of physicists) no solution to this problem is possible within the framework of conventional quantum mechanics" (Leggett, 1992, p. 231).

W. Zurek writes, “The key (and uncontroversial) fact has been known almost since the inception of quantum theory, but its significance . . . is being recognized only now: macroscopic systems are never isolated from their environment. Therefore they should not be expected to follow Schrödinger’s equation, which is applicable only to a closed system” (Zurek, 1991).

6. NOEFFECT $\not\Rightarrow$ PRODMARG

Consider the following measurement. A spin- $\frac{1}{2}$ particle **S** moving in the y -direction enters a Stern–Gerlach device oriented in the z -direction. In each output beam ($\pm z$) there is a SG device oriented in the x -direction. Detect **S** leaving one of the x -direction SG devices. Assign a value 0 to the measurement if **S** is detected in a $-x$ beam and a 1 if in a $+x$ beam. Then for every state of **S**, $\Pr(0) = \Pr(1) = \frac{1}{2}$. Think of this triple SG device as a fair coin tosser. The POVM $E_0 = E_1 = E_1 = \frac{1}{2}$ **I** represents the measurement: for every state σ of **S**, $\text{Tr}(E_0\sigma) = \text{Tr}(E_1\sigma) = \frac{1}{2}$.

Let **P** be another spin- $\frac{1}{2}$. Measure both **S** and **P** with triple SG devices. Absent any assumption about the joint measurement probabilities, we can imagine different POVMs giving those probabilities. One possibility is $E_{s \& p}$ with $E_{0 \& 0} = E_{0 \& 1} = E_{1 \& 0} = E_{1 \& 1} = \frac{1}{4} \mathbf{I} \otimes \mathbf{I}$. Another is $E'_{s \& p}$ with $E'_{0 \& 0} = E'_{1 \& 1} = \frac{1}{2} \mathbf{I} \otimes \mathbf{I}$ and $E'_{0 \& 1} = E'_{1 \& 0} = 0$. For every state of **S** + **P**, $E_{s \& p}$ predicts two *independent* fair coin tosses and $E'_{s \& p}$ predicts two *correlated* fair coin tosses, 0 with 0 and 1 with 1.

Straightforward calculations show that $E_{s \& p}$ satisfies both NOEFFECT and PRODMARG. From these, we can see that the JMF implies that $E_{s \& p}$ represents the joint measurement:

$$\begin{aligned} \Pr(s \& p) &= \text{Tr}[(E_s \otimes E_p)\tau] = \text{Tr}[(E_s \otimes \mathbf{I})(\mathbf{I} \otimes E_p)\tau] \\ &= \text{Tr}\left[\left(\sum_p E_{s \& p}\right)\left(\sum_s E_{s \& p}\right)\tau\right] = \text{Tr}(E_{s \& p}\tau). \end{aligned}$$

The POVM $E'_{s \& p}$ satisfies NOEFFECT but not PRODMARG. Thus NOEFFECT $\not\Rightarrow$ PRODMARG.

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REFERENCES

Busch, P., Lahti, P., and Mittelstaedt, P. (1991). *The Quantum Theory of Measurement*, Springer, Berlin.
 Cohen-Tannodji, C., Diu, B., and Laloë, F. (1977). *Quantum Mechanics*, Herman/Wiley, Paris.

- Davies, E. B. (1976). *Quantum Theory of Open Systems*, Academic Press, London.
- Jordan, T. (1983). Quantum correlations do not transmit signals. *Physics Letters A* **94**, 264.
- Kraus, K. (1983). *States, Effects, and Operations*, Springer, Berlin.
- Leggett, A. J. (1992). On the nature of research in condensed-state physics. *Foundation of Physics* **22**, 221.
- Macdonald, A. (2002). Quantum theory without measurement or state reduction problems. Available: <http://faculty.luther.edu/~macdonal>
- Mermin, D. (1981). Bringing home the atomic world. *American Journal of Physics* **49**, 940.
- Mermin, D. (1985). Is the moon there when nobody looks? *Physics Today* **38**(4), 38.
- Ozawa, M. (1997a). Quantum state reduction and the quantum bayes principle. In *Quantum Communication, Computing and Measurement*, O. Hirota *et al.*, eds., Plenum, New York. Also quant-ph/9705030.
- Ozawa, M. (1997a). An operational approach to quantum state reduction. *Annals of Physics* **259**, 121. Also quant-ph/9706027.
- Ozawa, M. (1998a). Quantum state reduction: An operational approach. *Fortschritte der Physics* **46**, 615. Also quant-ph/9711006.
- Ozawa, M. (1998b). On the concept of quantum state reduction: Inconsistency of the orthodox view, quant-ph/9802022.
- Ozawa, M. (2000a). Operational characterization of simultaneous measurements in quantum mechanics. *Physics Letters A* **275**, 5. This is a letter version of Ozawa, 2000c. A similar paper is available as quant-ph/9802039.
- Ozawa, M. (2000b). Measurements of nondegenerate discrete observables. *Physical Review A* **62**, 062101. Also quant-ph/0003033.
- Ozawa, M. (2000c). Operations, disturbance, and simultaneous measurability, quant-ph/0005054.
- Svetlichny, G. (2002). Causality implies formal state collapse, quant-ph/0207180.
- Zanchini, E. and Barletta, A. (1991). Absence of instantaneous transmission of signals in quantum theory of measurement. *Nuovo Cimento B* **106**, 419.
- Zurek, W. (1991). Decoherence and the transition from quantum to classical. *Physics Today* **44**(10), 36.